Chapter 4

MOVING CHARGES AND MAGNETIC FORCE

Magnetic effect of current

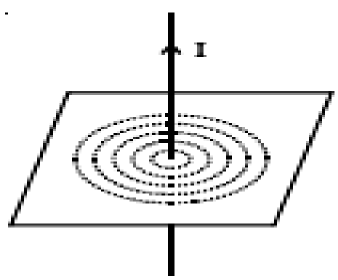
In 1820, Danish Physicist, Hans Christian Oersted observed that current through a wire caused a deflection in a nearby magnetic needle. This indicates that magnetic field is associated with a current carrying conductor.

Magnetic field around a straight conductor carrying current

A smooth cardboard with iron filings spread over it, is fixed in a horizontal plane with the help of a clamp. A straight wire passes through a hole made at the centre of the cardboard.

A current is passed through the wire by connecting its ends to a battery. When the cardboard is gently tapped, it is found that the iron filings arrange themselves along concentric circles.

This clearly shows that magnetic field is developed around a current carrying conductor.

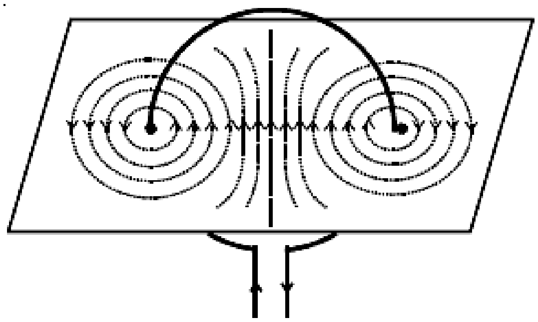


The direction of magnetic field with respect to the electric current through the conductor is given by ***Maxwells's right hand cork screw rule.***

It states that **if a right handed cork screw is rotated to advance along the direction of the current through a conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor**.

The direction of the magnetic field is given by Right-***hand thumb rule*** stated below:

Curl the fingers of right hand around the thumb, if the curled fingers give the direction of current, then right-hand thumb gives the direction of the magnetic field.



Magnetic field lines of a circular coil carrying current.

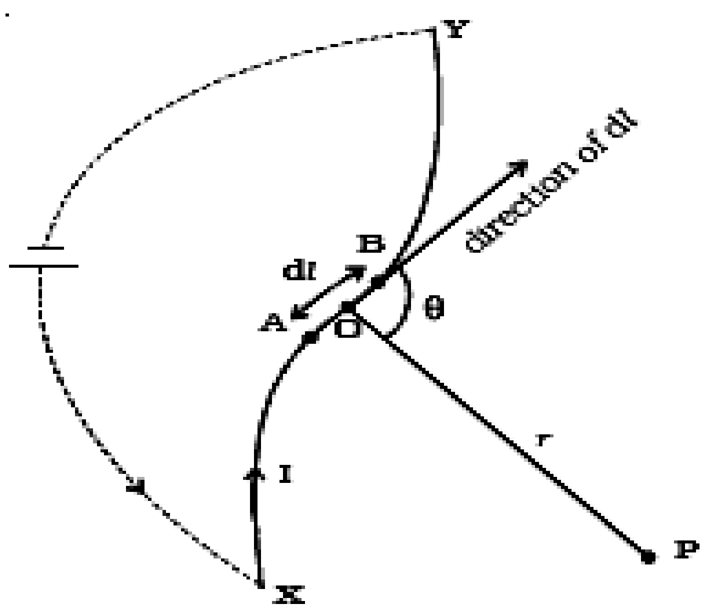
Biot – Savart Law

Biot and Savart conducted many experiments to determine the factors on which the magnetic field due to current in a conductor depends.

The results of the experiments are summarized as Biot-Savart law.

Let XY be a conductor carrying a current I

AB = d*l* is a small element of the conductor. P is a point at a distance r from the midpoint O of AB.



Biot - Savart law

The magnetic induction dB at P due
to the element of length dl is

(i) directly proportional to the current (I)

(ii) directly proportional to the length of the element (dl)

(iii) directly proportional to the sine of the angle between dl and the line joining element dl and the point P (sin θ)

(iv) inversely proportional to the square of the distance of the point from the element **(1/r²)**

$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

$$dB = K\,\frac{I\,dl\,\sin\theta}{r^2}$$

where

$$K = \frac{\mu}{4\pi}$$

$\mu = \mu_r\,\mu_o$ where μ_r is the relative permeability of the medium and
μ_0 is the permeability of free space.

$\mu_o = 4\pi \times 10^{-7}$ henry/metre. For air
$\mu_r = 1$.

So, in air medium

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,.\,dl\sin\theta}{r^2}$$

In vector form

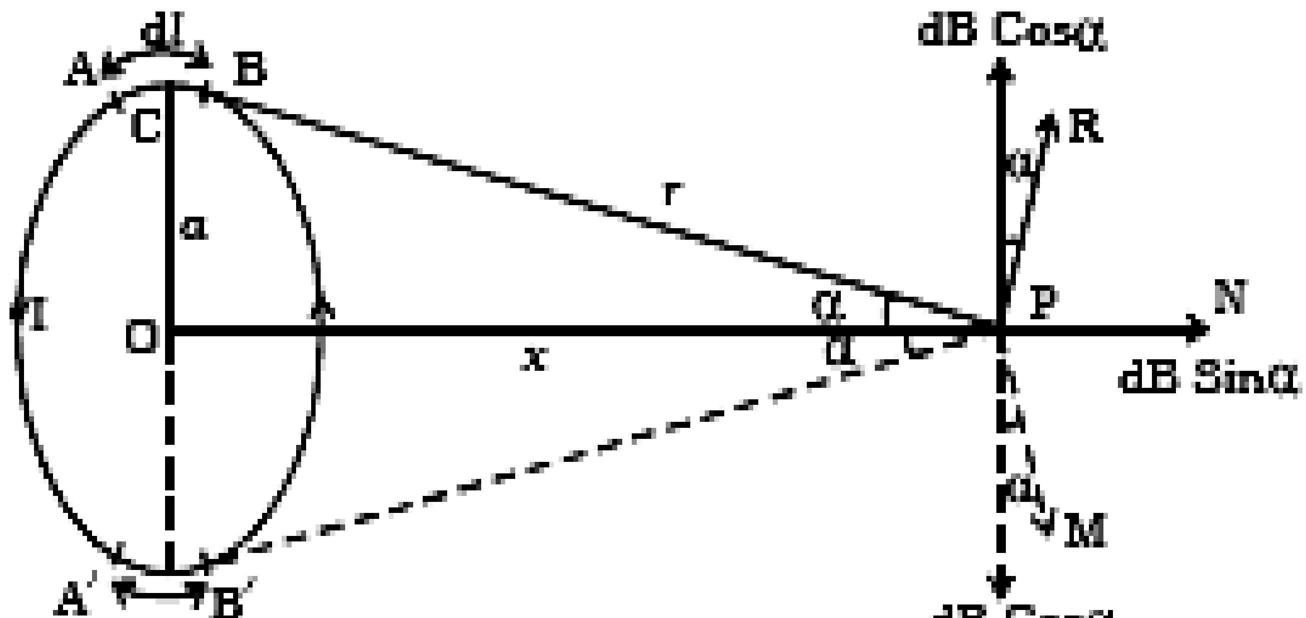
$$\overrightarrow{dB} = \frac{\mu_o}{4\pi}\frac{\overrightarrow{Idl}\times\bar{r}}{r^3} \qquad \text{or} \qquad \overrightarrow{dB} = \frac{\mu_o}{4\pi}\frac{\overrightarrow{Idl}\times\hat{r}}{r^2}$$

The direction of dB is perpendicular to the plane containing current element I d*l* and r (i.e plane of the paper) and acts inwards. The unit of magnetic induction is tesla (or) weber m^{-2}.

Application of Biot- Savart Law

Magnetic induction along the axis of a circular coil carrying current

Let us consider a circular coil of radius '*a*' with a current I as shown in Fig. P is a point along the axis of the coil at a distance *x* from the centre O of the coil.



AB is an infinitesimally small element of length d*l*. C is the midpoint of AB and CP = r .

According to Biot – Savart law, the magnetic induction at P due to the element d*l* is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2}$$

where θ is the angle between I dl and r $\theta = 90^{o}$.

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

The direction of dB is perpendicular to the current element I d*l* and CP. It is therefore along PR perpendicular to CP.

Considering the diametrically opposite element A'B', the magnitude of dB at P due to this element is the same as that for AB but its direction is along PM.

Let the angle between the axis of the coil and the line joining the element (d*l*) and the point (P) be α.

dB is resolved into two components :- dB sin α along OP and dB cos α perpendicular to OP.

dB cos α components due to two opposite elements cancel each other whereas dB sin α components get added up. So, the total magnetic induction at P due to the entire coil is

$$B = \int dB \sin\alpha$$

$$\int \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\frac{a}{r} = \frac{\mu_0}{4\pi}\frac{I a}{r^3}\int dl$$

If the coil contains *n* turns, the magnetic induction is

$$B = \frac{\mu_0 n I a^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

At the centre of the coil, $x = 0$

$$B = \frac{\mu_0 n I}{2a}$$

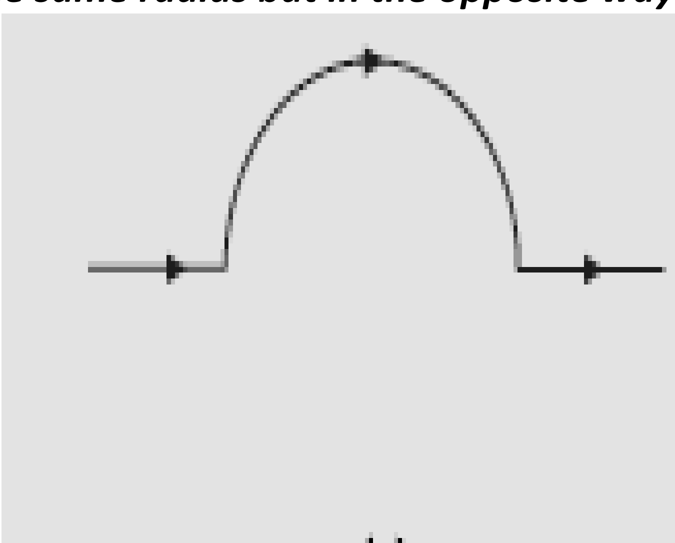
Text book example:

1. A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Fig. (a). Consider the magnetic field B at the centre of the arc

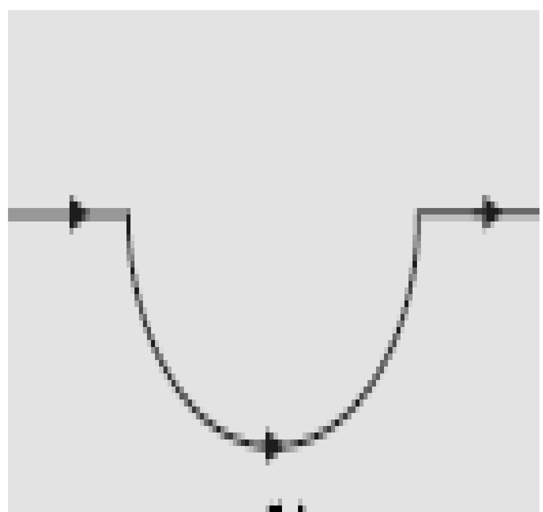
(a) What is the magnetic field due to the straight segments?

(b) In what way the contribution to B from the semicircle differs from that of a circular loop and in what way does it resemble?

(c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in Fig. b)?



(a)



(b)

Solution:

(a) d**l** and **r** for each element of the straight segments are parallel.
Therefore, d**l** × **r** = 0. Straight segments do not contribute to |**B**|.

(b) magnitude of B is half that of a circular loop. $B=\mu_0 I/4a$
Thus **B** is 1.9×10^{-4} T normal to the plane of the paper going into it.

(c) Same magnitude of **B** but opposite in direction to that in (a).

Ampere's Circuital Law

Biot – Savart law expressed in an alternative way is called Ampere's circuital law. The magnetic induction due to an infinitely long straight current carrying conductor is

$$B = \frac{\mu_o I}{2\pi a}$$

$$B\,(2\pi a) = \mu_o I$$

B ($2\pi a$) is the product of the magnetic field and the circumference of the circle of radius '*a*' on which the magnetic field is constant. If L is the perimeter of the closed curve and I_o is the net current enclosed by the closed curve, then the above equation may be expressed as,

$$\mathbf{BL = \mu_o I_o \;....(1)}$$

In a more generalized way, Ampere's circuital law is written as

$$\oint \vec{B}.\vec{dl} = \mu_o I_o$$

Ampere's circuital law states that "**The line integral of magnetic field for a closed curve is equal to μ_o times the net current I_o threading through the area bounded by the curve**".

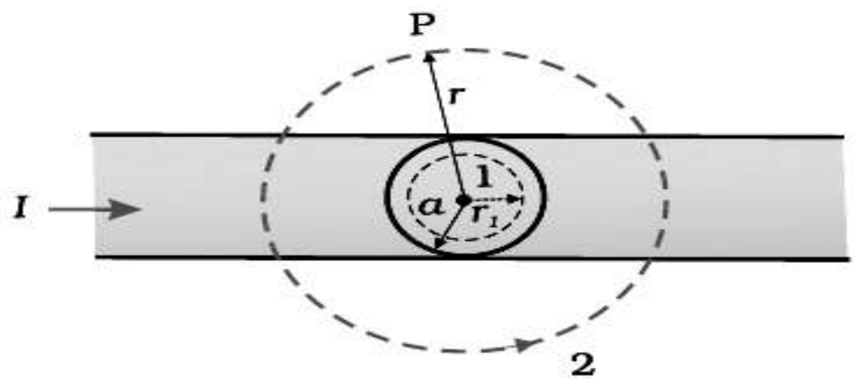
NOTE:

i) The line integral does not depend on the shape of the path or the position of the wire within the magnetic field.

ii) If the closed path does not encircle the current carrying wire (if a wire lies outside the path), the line integral of the field of that wire is zero.

Home Assignment:

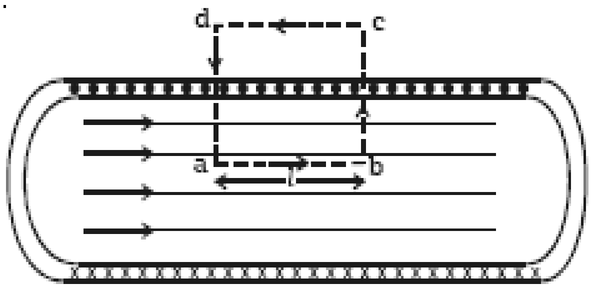
1. Figure shows a long straight wire of a circular cross-section (radius a) carrying steady current I. The current I is uniformly distributed across this cross-section. Calculate the magnetic field in the region r < a and r > a.



Refer Solution from NCERT Text book Pg.149, Example 4.8

Magnetic induction due to a long solenoid carrying current

Let us consider an infinitely long solenoid having *n* turns per unit length carrying a current of I. For such an ideal solenoid (whose length is very large compared to its radius), the magnetic field at points outside the solenoid is zero. A long solenoid appears like a long cylindrical metal sheet. The upper view of dots is like a uniform current sheet coming out of the plane of the paper. The lower row of crosses is like a uniform current sheet going into the plane of the paper.



To find the magnetic induction (B) at a point inside the solenoid. The line integral,

$$\oint \vec{B}.\vec{dl}$$

for the loop abcd is the sum of four integrals.

$$\therefore \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$$

If l is the length of the loop, the first integral on the right side is Bl. The second and fourth integrals are equal to zero because $\vec{B}$ is at right angles for every element $\vec{dl}$ along the path. The third integral is zero since the magnetic field at points outside the solenoid is zero.

$$\oint \vec{B}.\vec{dl} = Bl$$

Since the path of integration includes nl turns, the net current enclosed by the closed loop is

$$I_o = Inl$$

By Ampere's circuital law, equating the above two equations, we get

$$Bl = \mu_o \, Inl$$
$$B = \mu_o nI$$

The solenoid is commonly used to obtain uniform magnetic field. By inserting a soft iron core inside the solenoid, a large magnetic field is produced

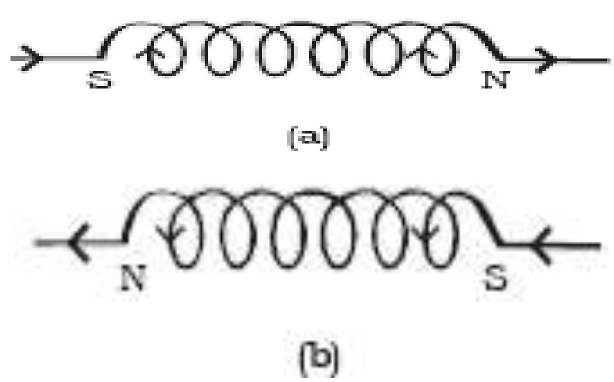
$$B = \mu nI = \mu_o \, \mu_r n \, I$$

when a current carrying solenoid is freely suspended, it comes to rest like a suspended bar magnet pointing along north-south. The magnetic polarity of the current carrying solenoid is given by **End rule.**

End rule

When looked from one end, if the current through the solenoid is along clockwise direction fig. a the nearer end corresponds to south pole and the other end is north pole.

When looked from one end, if the current through the solenoid is along anti-clock wise direction, the nearer end corresponds to north pole and the other end is south pole as in fig. b.

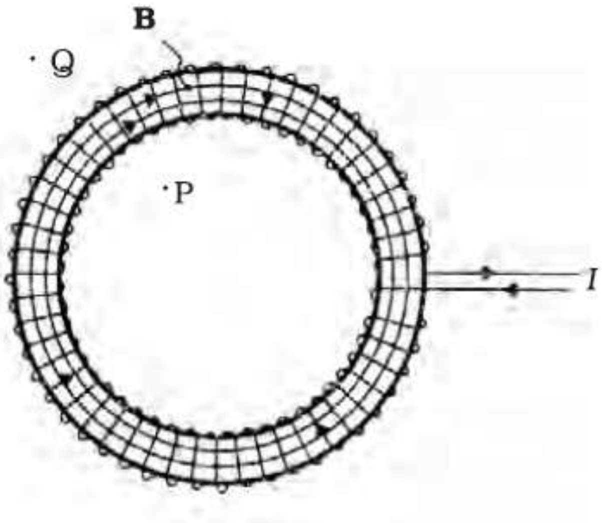


Class Assignment :

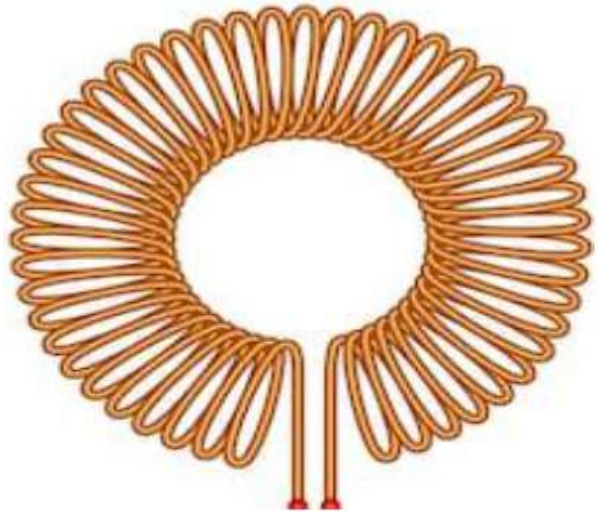
1. A solenoid is 2m long and 3 cm in diameter. It has 5 layers of windings of 1000 turns each and carries a current of 5A. Find the magnetic induction at its centre along its axis.

The toroid

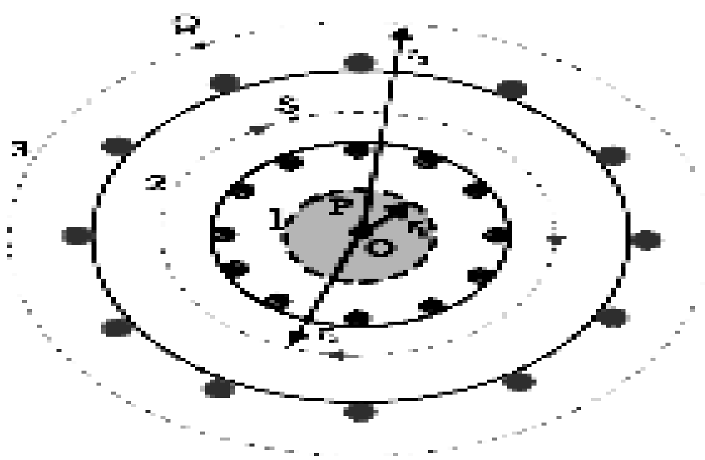
The toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. It can be viewed as a solenoid which has been bent into a circular shape to close on itself carrying a current I.



(a)



We shall see that the magnetic field in the open space inside (point P) and exterior to the toroid (point Q) is zero. The field B inside the toroid is constant in magnitude for the ideal toroid of closely wound turns.



(b)

The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops.

Three circular Amperian loops 1, 2 and 3 are shown by dashed lines. By symmetry, the magnetic field should be tangential to each of them and constant in magnitude for a given loop. The circular areas bounded by loops 2 and 3 both cut the toroid so that each turn of current carrying wire is cut once by the loop 2 and twice by the loop 3. Let the magnetic field along loop 1 be B_1 in magnitude.
Then in Ampere's circuital law $\mathbf{L = 2\pi\, r_1}$
However, the loop encloses no current, so

$$\mathbf{I_e = 0.\ Thus,\ B_1\,(2\,\pi\, r_1) = \mu_0(0),\ \ B_1 = 0}$$

Thus, the magnetic field at any point P in the open space inside the toroid is zero. The magnetic field at Q is likewise zero. Let the magnetic field along loop 3 be B_3. Once again from Ampere's law

$$\mathbf{L = 2\,\pi\, r_3.}$$

However, from the sectional cut, we see that the current coming out of the plane of the paper is cancelled exactly by the current going into it. Thus,

$$\mathbf{I_e = 0,\ and\ B_3 = 0}$$

Let the magnetic field inside the solenoid be B. We shall now consider the magnetic field at S. By Ampere's law ,

$$\mathbf{L = 2\pi\, r}$$

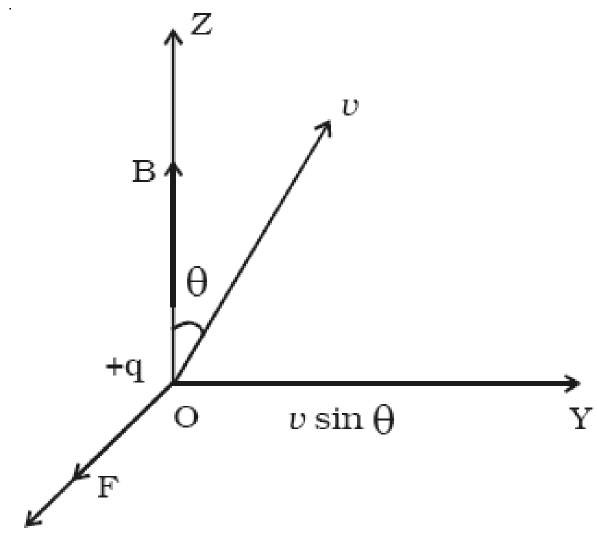
The current enclosed I_e is (for N turns of toroidal coil) = N I.

$$\mathbf{B\,(2\pi r) = \mu_0 NI}$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

Let r be the average radius of the toroid and n be the number of turns per unit length. Then N = 2πr n = (average) perimeter of the toroid × number of turns per unit length and thus, $\boldsymbol{B = \mu_0\, n\, I}$.

Magnetic Lorentz force



(a)

Let us consider a uniform magnetic field of induction B acting along the Z-axis. A particle of charge + q moves with a velocity *v* in YZ plane making an angle θ with the direction of the field

Under the influence of the field, the particle experiences a force F.

H.A.Lorentz formulated the special features of the force F (Magnetic Lorentz force) as under :

(i) the force F on the charge is zero, if the charge is at rest. (i.e) the moving charges alone are affected by the magnetic field.

(ii) the force is zero, if the direction of motion of the charge is either parallel or anti-parallel to the field and the force is maximum, when the charge moves perpendicular to the field.

(iii) the force is proportional to the magnitude of the charge (q)

(iv) the force is proportional to the magnetic induction (B)

(v) the force is proportional to the speed of the charge (v)

(vi) the direction of the force is opposite for charges of opposite sign.

All these results are combined in a single expression as

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

The magnitude of the force is

$$F = Bqv \sin\theta \,.$$

Since the force always acts perpendicular to the direction of motion of the charge, the force does not do any work.
In the presence of an electric field E and magnetic field B, the total force on a moving charged particle is

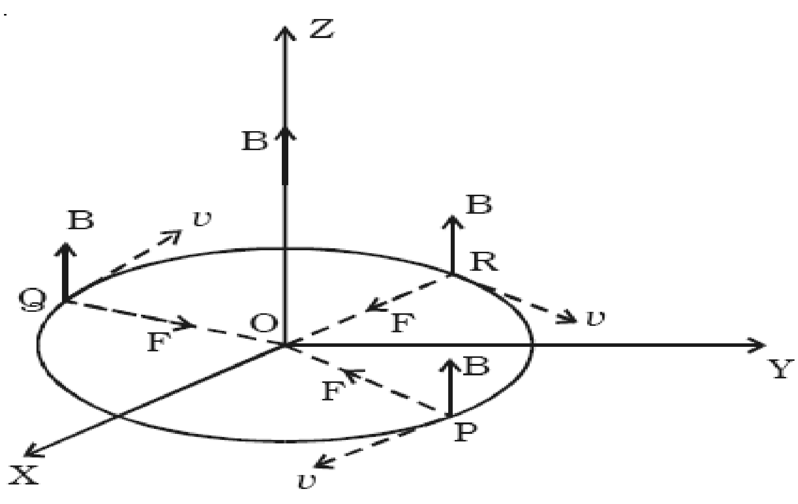
$$\vec{F} = q\,[(\vec{v} \times \vec{B}) + \vec{E}]$$

Motion of a charged particle in a uniform magnetic field.

Let us consider a uniform magnetic field of induction B acting along the Z-axis. A particle of charge *q* and mass *m* moves in XY plane. At a point P, the velocity of the particle is *v*. The magnetic lorentz force on the particle is

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

Hence $\vec{F}$ acts along PO perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.
Since the force acts perpendicular to its velocity, the force does not do any work. So, the magnitude of the velocity remains constant and only its direction changes. The force F acting towards the point O acts as the centripetal force and makes the particle to move along a circular path. At points Q and R, the particle experiences force along QO and RO respectively.



Since $\vec{v}$ and $\vec{B}$ are at right angles to each other F = Bqv sin 90^0 = Bqv.

This magnetic Lorentz force provides the necessary centripetal force.

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq} \qquad ...(1)$$

It is evident from this equation, that the radius of the circular path is proportional to (i) mass of the particle and (ii) velocity of the particle.

$$\frac{v}{r} = \frac{Bq}{m}$$

$$\omega = \frac{Bq}{m} \qquad ...(2)$$

This equation gives the angular frequency of the particle inside the magnetic field.

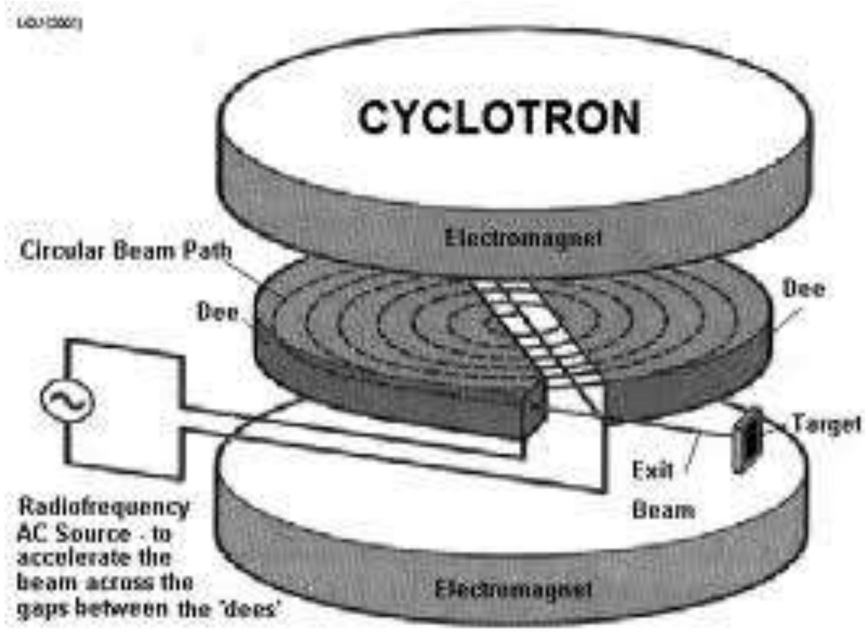
$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi m}{Bq} \qquad ...(3)$$

From equations (2) and (3), it is evident that the angular frequency and period of rotation of the particle in the magnetic field do not depend upon (i) the velocity of the particle and (ii) radius of the circular path.

Cyclotron

Cyclotron is a device used to accelerate charged particles to high energies. It was devised by Lawrence.

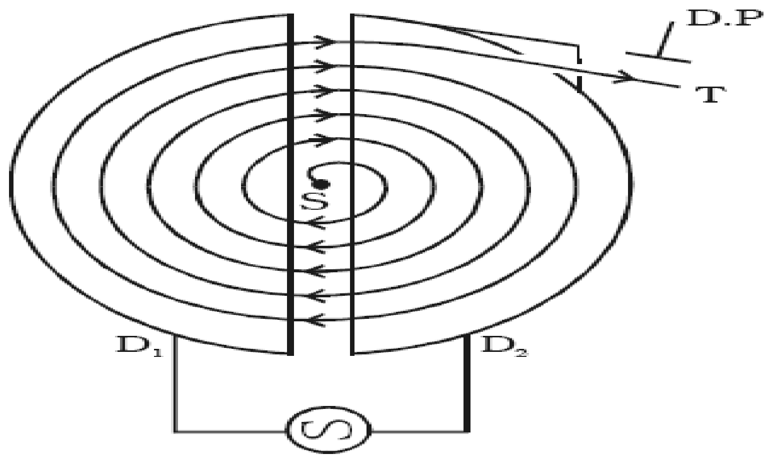


Principle

Cyclotron works on the principle that a charged particle moving normal to a magnetic field experiences magnetic Lorentz force due to which the particle moves in a circular path and its kinetic energy increases by acceleration due to high frequency AC. .

Construction

It consists of a hollow metal cylinder divided into two sections D_1 and D_2 called Dees, enclosed in an evacuated chamber. The Dees are kept separated and a source of ions is placed at the centre in the gap between the Dees. They are placed between the pole pieces of a strong electromagnet. The magnetic field acts perpendicular to the plane of the Dees. The Dees are connected to a high frequency oscillator.



Working

When a positive ion of charge q and mass m is emitted from the source, it is accelerated towards the Dee having a negative potential at that instant of time. Due to the normal magnetic field, the ion experiences magnetic Lorentz force and moves in a circular path. By the time the ion arrives at the gap between the Dees, the polarity of the Dees gets reversed. Hence the particle is once again accelerated and moves into the other Dee with a greater velocity along a circle of greater radius. Thus the particle moves in a spiral path of increasing radius and when it comes near the edge, it is taken out with the help of a deflector plate (D.P). The particle with high energy is now allowed to hit the target T.

When the particle moves along a circle of radius r with a velocity v, the magnetic Lorentz force provides the necessary centripetal force.

$$Bqv = \frac{mv^2}{r}$$

$$\frac{v}{r} = \frac{Bq}{m} = \text{constant} \quad \ldots(1)$$

The time taken to describe a semi-circle

$$t = \frac{\pi r}{v} \quad \ldots(2)$$

Substituting equation (1) in (2),

$$t = \frac{\pi m}{Bq} \quad \ldots(3)$$

It is clear from equation (3) that the time taken by the ion to describe a semi-circle is independent of (i) the radius (r) of the path and (ii) the velocity (v) of the particle.

Hence, period of rotation T = 2t

$$T = \frac{2\pi m}{Bq} = \text{constant} \qquad \ldots(4)$$

So, in a uniform magnetic field, the ion traverses all the circles in exactly the same time. The frequency of rotation of the particle,

$$\upsilon = \frac{1}{T} = \frac{Bq}{2\pi m} \qquad \ldots(5)$$

If the high frequency oscillator is adjusted to produce oscillations of frequency as given in equation (5), resonance occurs.

Kinetic energy of particle leaving Cyclotron

$$v = \frac{qBR}{m}$$

where v is the velocity, R is the radius of the trajectory at exit, and equals the radius of a dee.

Hence, the kinetic energy of the ions is,

$$\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

Use of Cyclotron:

Cyclotron is used to accelerate protons, deutrons and α - particles**.**

Limitations of Cyclotron

i)It cannot be used to accelerate Electron

ii) It cannot be used to accelerate Neutron

iii) Maintaining very large uniform magnetic field is not easier.

iv) At high velocities, relativistic variation of mass of the particle upsets the resonance condition.

Class Assignment:

2. Why Cyclotron cannot be used to accelerate Electron?

3. Why Cyclotron cannot be used to accelerate neutron?

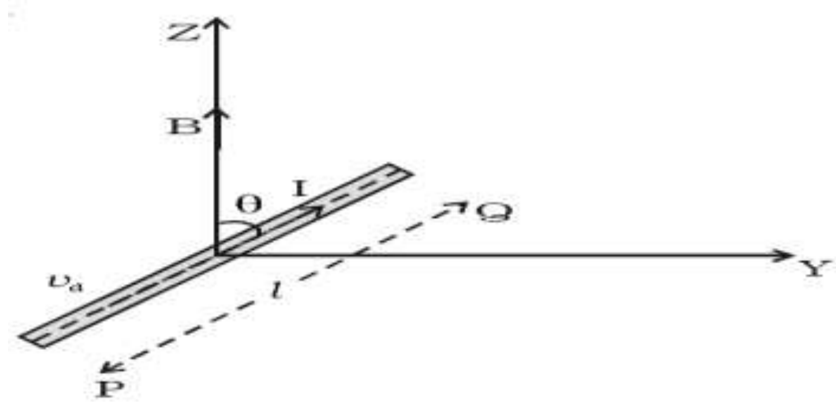
4. What will happen if the frequency of revolution of charged particle accelerated does not match the frequency of electric field applied?

5. An α-particle moves with a speed of 5×10^5 ms^{-1} at an angle of 30^o with respect to a magnetic field of induction 10^{-4} T. Find the force on the particle. [α particle has a +ve charge of 2e].

6. A stream of deutrons is projected with a velocity of 10^4 ms^{-1} in XY – plane. A uniform magnetic field of induction 10^{-3} T acts along the Z-axis. Find the radius of the circular path of the particle. (Mass of deuteron is 3.32×10^{-27} kg and charge of deuteron is 1.6×10^{-19} C).

Force on a current carrying conductor placed in a magnetic field.

Let us consider a conductor PQ of length *l* and area of cross section A. The conductor is placed in a uniform magnetic field of induction B making an angle θ with the field. A current I flows along PQ. Hence, the electrons are drifted along QP with drift velocity v_d.



If *n* is the number of free electrons per unit volume in the conductor, then the current is

$$\mathbf{I = nAv_de}$$

Multiplying both sides by the length *l* of the conductor,

$$\therefore \mathbf{Il = nAv_del.}$$

Therefore the current element,

$$\vec{Il} = -nA\vec{v_d}el \quad \ldots(1)$$

The negative sign in the equation indicates that the direction of current is opposite to the direction of drift velocity of the electrons. Since the electrons move under the influence of magnetic field, the magnetic lorentz force on a moving electron.

$$\vec{f} = -e\,(\vec{v_d} \times \vec{B}) \quad \ldots(2)$$

The negative sign indicates that the charge of the electron is negative.
The number of free electrons in the conductor

$$\mathbf{N = nAl} \quad \ldots(3)$$

The magnetic lorentz force on all the moving free electrons

$$\vec{\mathbf{F}} = \vec{\mathbf{Nf}}$$

Substituting equations (2) and (3) in the above equation

$$\vec{F} = nAl\,\{-e\,(\vec{v_d} \times \vec{B})\}$$

$$\vec{F} = -nAl\,e\,\vec{v_d} \times \vec{B} \quad \ldots(4)$$

Substituting equation (1) in equation (4)

$$\vec{F} = \vec{Il} \times \vec{B}$$

This total force on all the moving free electrons is the force on the current carrying conductor placed in the magnetic field.

Magnitude of the force

The magnitude of the force is F = BIl sin θ

(i) If the conductor is placed along the direction of the magnetic field, $\theta = 0^o$, Therefore force F = 0.

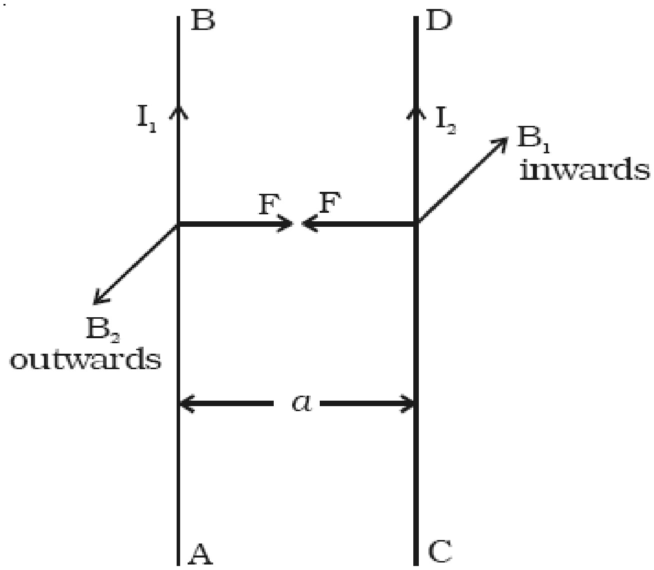
(ii) If the conductor is placed perpendicular to the magnetic field, $\theta = 90^o$, F = BIl. Therefore the conductor experiences maximum force.

Direction of force : The direction of the force on a current carrying conductor placed in a magnetic field is given by **Fleming's Left Hand Rule**.

The forefinger, the middle finger and the thumb of the left hand are stretched in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, the middle finger points in the direction of the current, then the thumb points in the direction of the force on the conductor.

Force between two long parallel current-carrying conductors

AB and CD are two straight very long parallel conductors placed in air at a distance a. They carry currents I_1 and I_2 respectively.



The magnetic induction due to current I_1 in AB at a distance a is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \qquad ...(1)$$

This magnetic field acts perpendicular to the plane of the paper and inwards. The conductor CD with current I_2 is situated in this magnetic field. Hence, force on a segment of length l of CD due to magnetic field B_1 is

$$F = B_1 I_2 l$$

substituting equation (1)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \qquad ...(2)$$

By Fleming's Left Hand Rule, F acts towards left. Similarly, the magnetic induction due to current I2 flowing in CD at a distance a is

$$B_2 = \frac{\mu_o I_2}{2\pi a} \quad \ldots(3)$$

This magnetic field acts perpendicular to the plane of the paper and outwards. The conductor AB with current I_1, is situated in this field. Hence force on a segment of length l of AB due to magnetic field B_2 is

$$F = B_2 I_1 l$$

substituting equation (3)

$$F = \frac{\mu_o I_1 I_2 l}{2\pi a} \quad \ldots(4)$$

By Fleming's left hand rule, this force acts towards right. These two forces given in equations (2) and (4) attract each other. Hence, two parallel wires carrying currents in the same direction attract each other and if they carry currents in the opposite direction, repel each other.

Definition of ampere

The force between two parallel wires carrying currents on a segment of length l is

$$F = \frac{\mu_o I_1 I_2}{2\pi a} l$$

∴ Force per unit length of the conductor is

$$\frac{F}{l} = \frac{\mu_o I_1 I_2}{2\pi a}$$

If I1 = I2 = 1A, a = 1m

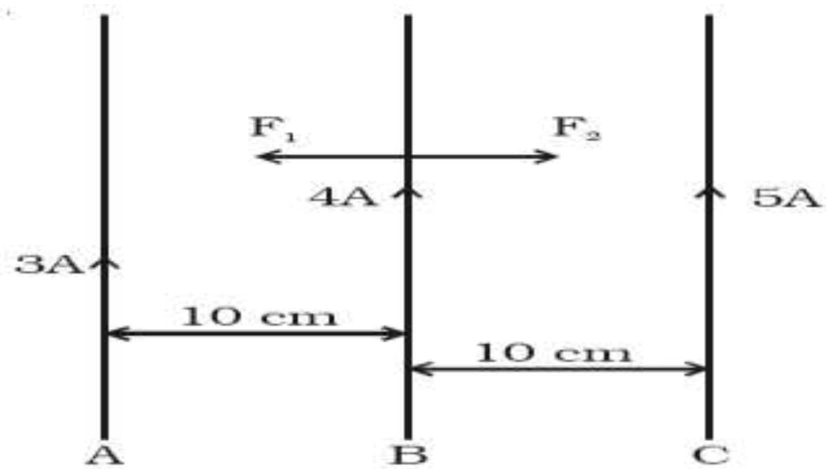
$$\frac{F}{l} = \frac{\mu_o}{2\pi} \frac{1 \times 1}{1} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ Nm}^{-1}$$

The above conditions lead the following definition of ampere.

Ampere is defined as that constant current which when flowing through two parallel infinitely long straight conductors of negligible cross section and placed in air or vacuum at a distance of one metre apart, experience a force of 2×10^{-7} newton per unit length of the conductor.

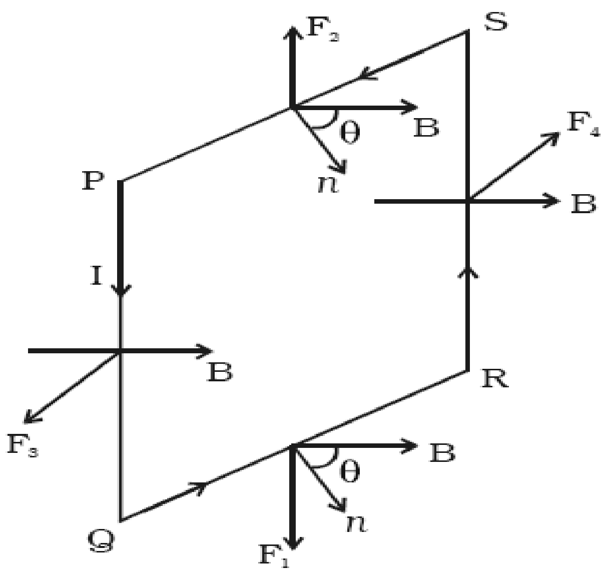
Class Assignment:

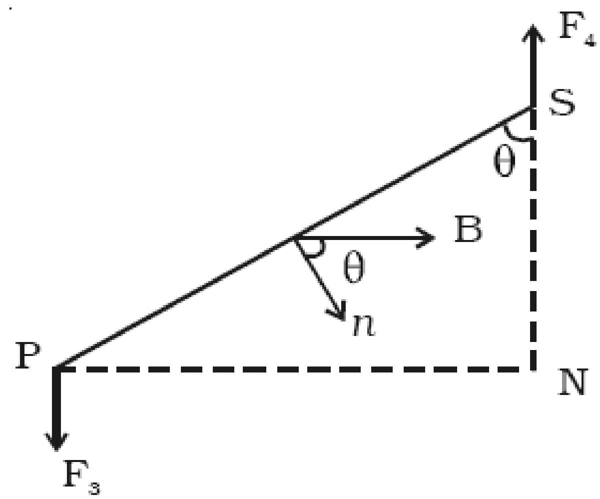
7. A, B and C are three parallel conductors each of length 10 m, carrying currents as shown in the figure. Find the magnitude and direction of the resultant force on the conductor B.



Torque experienced by a current loop in a uniform magnetic field

Let us consider a rectangular loop PQRS of length *l* and breadth *b*. It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction B. Let θ be the angle between the normal to the plane of the loop and the direction of the magnetic field.





Force on the arm QR,

$$\vec{F_1} = \overline{I(QR)} \times \vec{B}$$

Since the angle between

$$\overline{I(QR)} \text{ and } \vec{B} \text{ is } (90^0 - \theta),$$

Magnitude of the force

$$F_1 = BIb \sin(90^o - \theta)$$
$$\text{ie. } F_1 = BIb \cos\theta$$

Force on the arm SP,

$$\vec{F_2} = \overline{I(SP)} \times \vec{B}$$

Since the angle between

$$\overline{I(SP)} \text{ and } \vec{B} \text{ is } (90^o + \theta).$$

Magnitude of the force F_2 = BIb cos θ
The forces F1 and F2 are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is zero.
Force on the arm PQ,

$$\vec{F}_3 = \overrightarrow{I(PQ)} \times \vec{B}$$

Since the angle between

$$\overrightarrow{I(PQ)} \text{ and } \vec{B} \text{ is } 90^o,$$

Magnitude of the force F3 = BIl sin 90o = BIl
F3 acts perpendicular to the plane of the paper and outwards.
Force on the arm RS,

$$\vec{F}_4 = \overrightarrow{I(RS)} \times \vec{B}$$

Since the angle between

$$\overrightarrow{I(RS)} \text{ and } \vec{B} \text{ is } 90^o,$$

Magnitude of the force F4 = BIl sin 90^o = BIl

F_4 acts perpendicular to the plane of the paper and inwards. The forces F_3 and F_4 are equal in magnitude, opposite in direction and have different lines of action. So, they constitute a couple.
Hence, Torque τ = BIl × PN

= BIl × PS × sin θ

= BIl × b sin θ = BIA sin θ

If the coil contains n turns, τ = nBIA sin θ
So, the torque is **i)** maximum when the coil is parallel to the magnetic field and **ii)** zero when the coil is perpendicular to the magnetic field.
We define the magnetic moment of the current

loop as, **M**=n I **A** where the direction of the area vector **A** is given by the right-hand thumb rule and is directed into the plane of the paper. Then as the angle between **M** and **B** is θ , torque in vector form

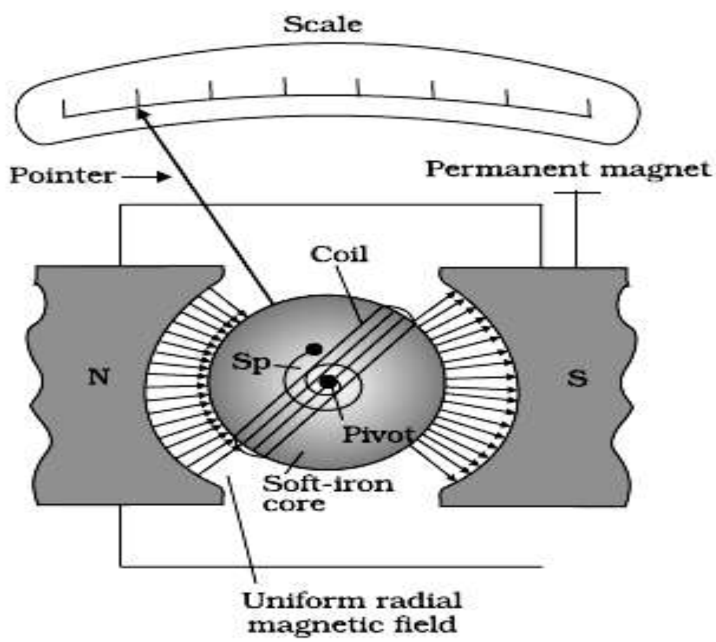
$$\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}$$

THE MOVING COIL GALVANOMETER

Moving coil galvanometer (MCG), is a device whose principle can be understood on the basis of our discussion in previous section.

The galvanometer consists of a coil, with many turns, free to rotate about a fixed axis in a uniform radial magnetic field.

There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field.



When a current flows through the coil, a torque acts on it. This torque is given by

$$\tau = NI\,AB \sin\theta$$

$$= NIAB$$

where the symbols have their usual meaning. Since the field is radial by design, we have taken sin θ = 1 in the above expression for the torque.

The magnetic torque NIAB tends to rotate the coil. A spring Sp provides a counter torque kϕ that balances the magnetic torque NIAB; resulting in a steady angular deflection ϕ. In equilibrium

$$k\phi = NI\,AB$$

where k is the torsional constant of the spring; i.e. the restoring torque per unit twist.

The deflection ϕ is indicated on the scale by a pointer attached to the spring.

$$\phi = \left(\frac{NAB}{k}\right)I$$

Current sensitivity of a galvanometer is defined as the deflection per unit current.

$$\frac{\phi}{I} = \frac{NAB}{k}$$

unit A/div

The **current sensitivity** of a galvanometer can be **increased** by

(i) increasing the number of turns

(ii) increasing the magnetic induction

(iii) increasing the area of the coil

(iv) decreasing the couple per unit twist of the suspension wire.

This explains why phosphor-bronze wire is used as the suspension wire ,since it has small couple per unit twist.

Voltage sensitivity of a galvanometer

The voltage sensitivity of a galvanometer is defined as the deflection per unit voltage.

$$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V}$$
$$= \left(\frac{NAB}{k}\right)\frac{1}{R}$$

where R is the galvanometer resistance

increasing the current sensitivity does not necessarily, increase the voltage sensitivity. For example ,when the number of turns (n) is doubled, current sensitivity is also doubled.

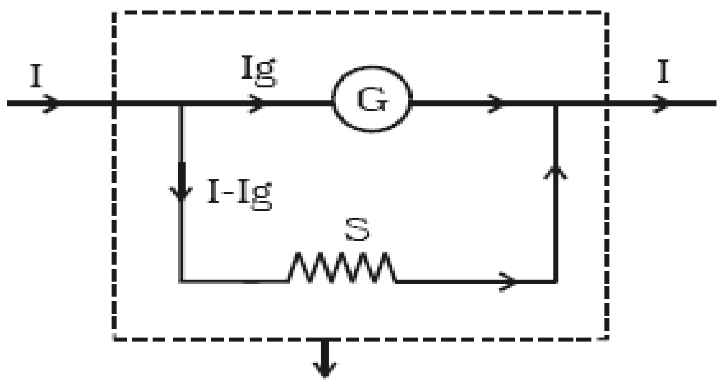
But increasing the number of turns correspondingly increases the resistance R. Hence voltage sensitivity remains unchanged.

Conversion of galvanometer into an ammeter

A galvanometer is a device used to detect the flow of current in an electrical circuit.

Being a very sensitive instrument, a large current cannot be passed through the galvanometer, as it may damage the coil.

A galvanometer is converted into an ammeter by connecting a low resistance S called shunt resistance in parallel with it.



As a result, when large current flows in a circuit, only a small fraction of the current passes through the galvanometer and the remaining larger portion of the current passes through the shunt.

The scale is marked in ampere.

Let Ig be the maximum current that can be passed through the galvanometer. The current Ig will give full scale deflection in the galvanometer.

Galvanometer resistance = G

Shunt resistance = S

Current in the circuit = I

∴ Current through the shunt resistance = $I_s = (I – I_g)$

Since the galvanometer and shunt resistance are parallel, potential is common.

∴ Ig G = (I- Ig)S

$$S = G \frac{I_g}{I - I_g}$$

The effective resistance of the ammeter Ra is

$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S}$$

$$R_a = \frac{GS}{G + S}$$

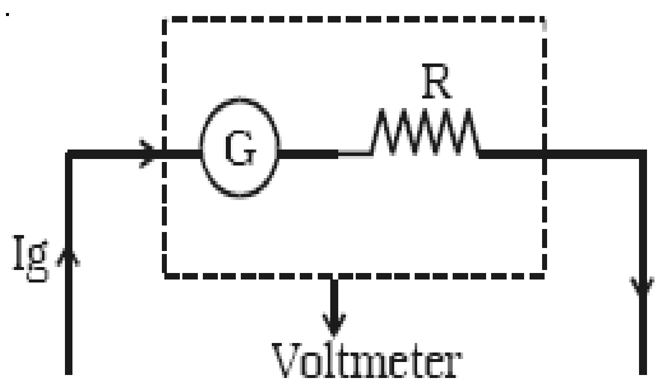
Ra is very low and, when connected in series, the ammeter does not appreciably change the resistance and current in the circuit.

An ideal ammeter is one which has almost zero resistance.

Conversion of galvanometer into a voltmeter

Voltmeter is an instrument used to measure potential difference between the two ends of a current carrying conductor.

A galvanometer can be converted into a voltmeter by connecting a high resistance R in series with it.



The scale is calibrated in volt. The value of the resistance connected in series decides the range of the voltmeter.

Galvanometer resistance = G

The current required to produce full scale deflection in the galvanometer = Ig

Range of voltmeter = V

Resistance to be connected in series = R

Since R is connected in series with the galvanometer, the current through the galvanometer,

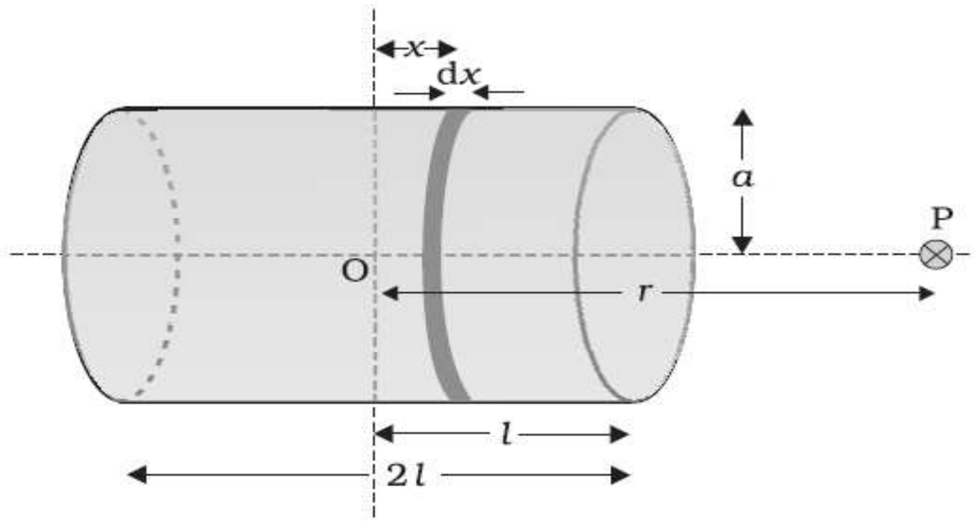
$V = I_g (R+G)$

$$R = \frac{V}{I_g} - G$$

The effective resistance of the voltmeter is
$R_v = G + R$
Rv is very large, and hence a voltmeter is connected in parallel in a circuit as it draws the least current from the circuit. So that pd across the part of the circuit is equal to voltmeter reading
An **ideal voltmeter** is one which has infinite resistance.

Class assignment
8. Mention one use of shunt resistance.
9. Why ammeter is always connected in series ?
10. Define ideal ammeter.
11. Why a voltmeter is always connected in parallel?
12. Define ideal voltmeter.
13. What is the advantage of radial magnetic field in a moving coil galvanometer?
14. What are the advantage s of cylindrical soft iron core in a MCG ?

Bar magnet as an equivalent solenoid
The magnetic dipole moment **m** associated with a current loop was defined to be **m** = *NI* **A** where *N* is the number of turns in the loop, *I* the current and **A** the area vector. The resemblance of magnetic field lines for a bar magnet and a solenoid suggest that
a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid.



Let the solenoid of Fig. consists of n turns per unit length. Let its length be $2l$ and radius a. We can evaluate the axial field at a point P, at a distance r from the centre O of the solenoid. To do this, consider a circular element of thickness dx of the solenoid at a distance x from its centre. It consists of $n\,d\,x$ turns. Let I be the current in the solenoid.

the magnitude of the field at point P due to the circular element is

$$dB = \frac{\mu_0 n\, dx\, I a^2}{2[(r-x)^2 + a^2]^{3/2}}$$

The magnitude of the total field is obtained by summing over all the elements — in other words by integrating from x = – l to x = + l.

Consider the far axial field of the solenoid, i.e.,

r >> a and r >> l . Then the denominator is approximated by

$$[(r-x)^2 + a^2]^{3/2} \approx r^3$$

$$B = \frac{\mu_0 n I a^2}{2r^3} \int_{-l}^{l} dx$$

$$= \frac{\mu_0 n I}{2} \frac{2 l a^2}{r^3}$$

Note that the magnitude of the magnetic moment of the solenoid is,

$$m = n\,(2l)\,I\,(\pi a^2)$$

(total number of turns × current × cross-sectional area). Thus,

$$B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

This is also the magnetic for axial magnetic field of a bar magnet which one may obtain experimentally. Thus, a bar magnet and a solenoid produce similar magnetic fields.

Circular current loop as a magnetic dipole

The magnetic field (at large distances) due to current in a circular current loop is very similar in behavior to the magnetic field of a magnetic dipole. The magnitude of magnetic field on the axis of a circular loop, of a radius *R*, carrying a steady current I is

$$B = \frac{\mu_0 I\,R^2}{2\left(x^2 + R^2\right)^{3/2}} \qquad \textbf{... (1)}$$

x is the distance along the axis from the centre of the loop. For *x* >> *R*, we may drop the *R*2 term in the denominator. Thus,

$$B = \frac{\mu_0 R^2}{2x^3} \qquad \textbf{...(2)}$$

the area of the loop A = πR^2. Thus,
multiplying and dividing RHS by π

$$B = \frac{\mu_0 IA}{2\pi x^3} \qquad \textbf{...(3)}$$

we define the magnetic moment **m** to have a magnitude IA, **m** = I **A**. Hence,

$$\mathbf{B} = \frac{\mu_0 \mathbf{m}}{2\pi x^3}$$

$$= \frac{\mu_0}{4\pi}\frac{2\mathbf{m}}{x^3} \quad ...(4)$$

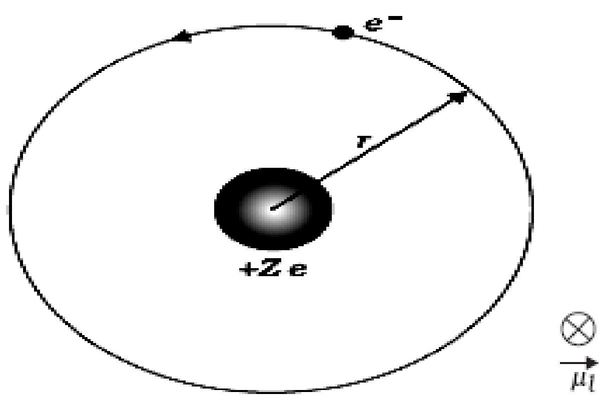
The magnetic field at any point due a short magnetic dipole along its axis is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}}{x^3}; \quad ... (5)$$

From equation (4) and (5) we find circular current loop is very similar in behavior to the magnetic dipole.

The magnetic dipole moment of a revolving electron

In the Bohr model, the electron (a negatively charged particle) revolves around a positively charged nucleus .The electron of charge (–e) revolves around a stationary heavy nucleus of charge +Ze.



This constitutes a current I,

$$I = \frac{e}{T}$$

T is the time period of revolution. Let r be the orbital radius of the electron, and v the orbital speed. Then

$$T = \frac{2\pi r}{v}$$

substituting for T in I we have

$$I = ev/2\pi r$$

A magnetic moment μ_l , associated with this circulating current in magnitude is, **μ_l = Iπr^2 = evr/2**. The direction of this magnetic moment is into the plane of the paper.

Multiplying and dividing the right-hand side of the above expression by the electron mass m_e,

$$\mu_l = \frac{e}{2m_e}(m_e v r)$$
$$= \frac{e}{2m_e} l$$

l is the magnitude of the angular momentum of the electron about the central nucleus ("orbital" angular momentum). Vectorially

$$\boldsymbol{\mu}_l = -\frac{e}{2m_e}\mathbf{l}$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment. If we had taken a particle with charge (+q), the angular momentum and magnetic moment would be in the same direction.

The ratio

$$\frac{\mu_l}{l} = \frac{e}{2m_e}$$

is called the **gyromagnetic ratio** and is a constant. Its value is 8.8×10^{10} C /kg for an electron.

Bohr hypothesized that the angular momentum assumes a discrete set of values, namely,

$$l = \frac{nh}{2\pi}$$

where n is a natural number, n = 1, 2, 3, ... and h is a constant named after Max Planck (Planck's constant) with a value h = 6.626×10^{-34} J s.
For n =1 in the above equation,

$$(\mu_l)_{min} = \frac{e}{4\pi m_e} h$$

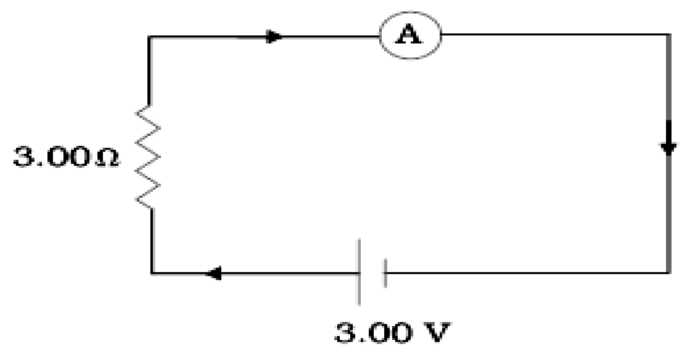
$$= \frac{1.60 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31}}$$

$$= 9.27 \times 10^{-24} \text{ Am}^2$$

This value is called the **Bohr magneton**.
Any charge in uniform circular motion would have an associated magnetic moment given by an expression similar to this dipole moment is labelled as the orbital magnetic moment. Hence the subscript 'l' in μ_l. Besides the orbital moment, the electron has an intrinsic magnetic moment, which has the same numerical value as given in above equation. It is called the spin magnetic moment. The microscopic roots of magnetism in iron and other materials can be traced back to this intrinsic spin magnetic moment.

Text book numerical

1. In the circuit the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance G = 60.00 Ω; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance S = 0.02 Ω; (c) is an ideal ammeter with zero resistance?



Solution:

(a) Total resistance in the circuit is,

$$G + 3 = 63\ \Omega.$$

$$I = 3/63 = 0.048\ \text{A}$$

(b) Resistance of the galvanometer converted to an ammeter is,

$$(G \times S) / (G+S)$$

$$= \frac{60\ \ \Omega \times 0.02\Omega}{(60+0.02)\Omega} = 0.02\,\Omega$$

Total resistance in the circuit is,

$$0.02\Omega + 3\Omega = 3.02\Omega$$

$$I = 3/3.02 = 0.99\ \text{A}.$$

(c) For the ideal ammeter with zero resistance,

$$I = 3/3 = 1.00\ \text{A}$$

Textbook Questions:

1. A long straight wire carries a current of 35 A. What is the magnitude of the field B at a point 20 cm from the wire?

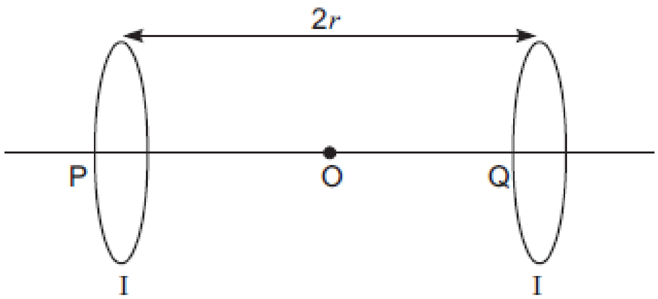
2. A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of B at a point 2.5 m east of the wire.

3. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 18.0 A, estimate the magnitude of B inside the solenoid near its centre.

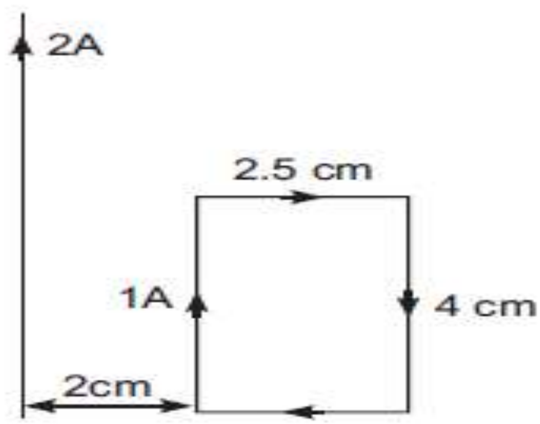
4. A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an

angle of 30º with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

5. Two identical circular loops, P and Q, each of radius r and carrying equal currents are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O which is equidistant from the loops P and Q.
Find the magnitude of the net magnetic field at O.



6. A rectangular loop of wire of size 2.5 cm × 4 cm carries a steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and the wire are coplanar, find the (i) torque acting on the loop and (ii) the magnitude and direction of the force on the loop due to the current carrying wire.



7.Two identical circular wires P and Q each of radius R and carrying current 'I' are kept in perpendicular planes such that they have a common centre as shown in

the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils.